

Improvement of Wideband Spectrum sensing Reconstruction Algorithms for Wireless Networks

T V N L Aswini, Padma Raju. K, Leela Kumari. B

Abstract: There is a tremendous growth in wireless networks and services from the last few years, which increased the urge of radio spectrum. This paved way for the new technology named Cognitive Radio which provides a promising solution for efficient spectrum utilization. Of all the different works of CR, sensing plays a vital role. In this aspect, Compressive sensing, a new paradigm joined hands in further improving the efficiency of CR by sampling the wideband spectrum at sub-Nyquist rates. In this piece of work, proposed enhanced Orthogonal matching pursuit algorithm proved its advantages over normal OMP algorithm by extending the iterations with a run factor. Simulation results justified the increase of detection probability with the proposed algorithm.

Index Terms: MWC, spectrum sensing, OMP, support.

I. INTRODUCTION

In the current scenario, there is an extraordinary rise for the demand of wireless devices and networks. This sudden increase of demand led to various wireless applications in all the areas. According to IEEE standards, government agencies allotted certain band of frequencies fixed to various wireless services. But all the time, the allotted fixed spectrum may not be in use. This results in inefficient usage of spectrum. Hence less usage and lack of radio spectrum problem made the wireless users search for the effective solution. **Cognitive Radio (CR)** is a new technology which senses the available radio spectrum intelligently. It senses the vacant spectrum and allocates intelligently to the secondary users temporarily when it is not being used by primary users, thus utilizing the spectrum efficiently. If at meantime, the primary users are back to use the spectrum, it leaves for the legacy users and mobilizes for other vacant band. Thus CR effectively utilizes the wireless spectrum. Of all the works of CR, sensing plays an important role. As the spectrum is wideband, it requires higher sampling rates which cannot be affordable even by today's best ADCs and also requirement of multiple functional blocks still increases the hardware complexity effecting the power consumption and speed. To overcome these problems, efficient sensing methods are required. Donoho [1] proposed a new framework named Compressive sensing to sense the wideband spectrum which speed up the acquisition process and reduces the implementation costs. Traditional approach of sampling is done at Nyquist rates defined by Shannon.

As the spectrum is a wide band, the Nyquist sampling is at very high rates which cannot be affordable by normal ADCs and the hardware complexity increases. The compressive sensing mechanism framed a new era by compressing and sensing the signal at a time. In this technique, sampling is done at sub-Nyquist rates. As shown in Fig.1, the signal of interest is a K-sparse signal $x(t)$ of length $N \times 1$ is sampled by obtaining the measurement matrix of $M \times N$ samples, then finding the M measurements gives the compressed samples. Finally, various reconstruction algorithms help to recover the original signal.

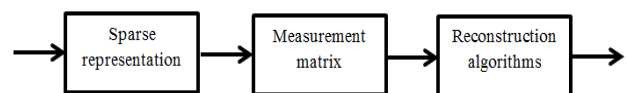


Fig. 1 Compressive sensing architecture

The wideband spectrum is a multiband signal was considered which spreads at continuous intervals over the spectrum. As wideband signal is sparse, sampling can be achieved through compressive sensing phenomenon. Several techniques are proposed to replace conventional ADCs. Landau [2] proposed a sensing method done at low rate sampling with exact recovery of the signal. Random demodulator proposed by Tropp [3] is a single channel acquisition scheme. Later [4] Moshe mishali developed modulated wideband converter by modifying random demodulator introducing the parallel channel structure. The main objective of this work is to provide a unified framework for sensing wideband spectrum at subnyquist rates and also faster recovery. The total spectrum is divided into slices and the energy in each slice reflects the information of vacant bands. The greedy pursuit algorithms provide fast recovery, also suitable for reconstruction in CS technique. Various derivatives of OMP [7] like Regularised OMP, Stagewise OMP, Adaptive OMP, etc., are derived as reconstruction algorithms which proved their performance over BP. However, enhanced OMP proposed here for MWC has sustained its advantages over OMP. Organization of the paper is as follows: a sparse multiband model is outlined in section II. Section III describes the subnyquist sampling scheme. In section IV, reconstruction algorithms and its extended versions are described. Numerical and simulation results are given in section V.

II. PROBLEM STATEMENT

Let a multiband model of $x(t)$ which has its spectrum spread across wide frequency range such that $X(f) = 0, |f| > \frac{f_{NYQ}}{2}$. It has N bands with a band width

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B Hz each. If B_i is the bandwidth of i^{th} band, then the symmetric bands should satisfy $B_i \leq B$. Hence the sampling rate is $NB \ll f_{NYQ}$ which indicate that the spectrum is not used at all times as shown if Fig.2. These frequency bands are said to be in use and the remaining spectrum is free called spectrum holes. CR detects these spectrum holes and allots to the secondary users. An algorithm is needed to detect the spectral support and signal reconstruction at faster rates with less hardware complexity. Of all the digital algorithms proposed, MWC poses greater advantages for spectrum sensing.

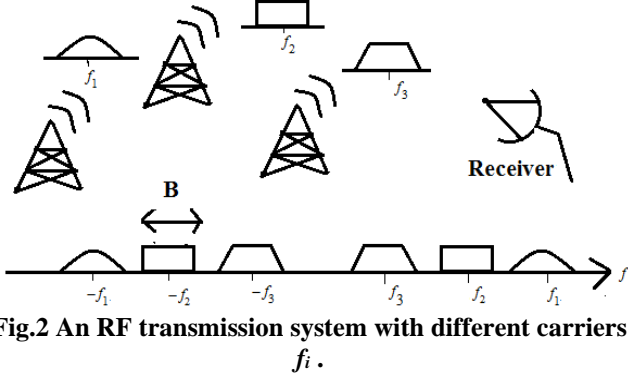


Fig.2 An RF transmission system with different carriers f_i .

A. Wideband Sensing model - MWC

The architecture of Modulated Wideband Converter (MWC) comprises of a group of modulators (mixers) and low pass filters. It is multi-channel SubNyquist sampling scheme consists of m parallel channels. At every channel, the input multiband signal $x(t)$ is mixed with a pseudo-random sequence $p_i(t)$ given by Eq.1 as

$$p_i(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_p n t} \quad (1)$$

And the coefficients

$$C_n = \frac{1}{2\pi} \left(\sum_{k=0}^{M-1} \alpha_{ik} e^{-j\frac{2\pi}{M}nk} \right) \quad (2)$$

It is a periodic sequence of period T_p having M number of values of $\alpha_{ik} = [+1, -1]$ with frequency $f_p \geq B$. Multiplication of pseudorandom sequence with input signal is given as $x_m(t) = p_i(t)x(t)$. The signals get convolved in frequency domain as $X_m(f) = P_i(f) * X(f)$. This process of mixing spreads the spectrum [8] and provides f_p shifted replicas of $X(f)$. Then low pass filtered with cut-off frequency of $f_c = f_s / 2$, distributes the overlaid energy across baseband.

After low pass filtering the signal, a low rate sampler is used to sample $y_i[n]$ at all the channels $0,1,\dots,m$. Since there are m channels, the total sampling rate is $mf_s = \frac{m}{N} f_{NYQ}$. In matrix form written as $\mathbf{y} = \mathbf{Cz}$,

where \mathbf{C} is the coefficient matrix and \mathbf{z} is the replicated vector of input signal. Its Fourier transform is given as

$$Y_i(f) = X_m(f) * H(f) = \sum_{n=-L_0}^{L_0} C_n X(f - nf_p) H(f)$$

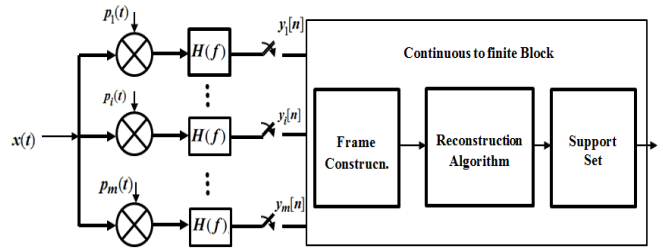


Fig.3. Modulated wideband converter architecture

B. Continuous-To-Finite (CTF) block

Signal reconstruction takes place in Continuous-To-Finite (CTF) block which performs support recovery. It constructs a basis or frame from the measurements such as $Q = \mathbf{V}\mathbf{V}^H$. The underdetermined system of $\mathbf{V}=\mathbf{A}\mathbf{U}$ provides the occupied spectrum information [9,10,11]. A unique solution to find the support set \mathbf{S} is done by using any reconstruction algorithm. This complete architecture of MWC is depicted in Fig.3. At this point the CR knows the information of vacant bands to allocate secondary users. The support set is a column vector having k non-zero indices. If the support set \mathbf{S} is known, \hat{x} can be recovered by using a submatrix \mathbf{A}_s constructed by columns of \mathbf{A} specified by vector \mathbf{S} . Thus reconstructed signal is obtained as

$$\hat{x} = \left(\mathbf{A}_s^* \mathbf{A}_s \right)^{-1} \mathbf{A}_s^* \mathbf{V} \quad (3)$$

where \mathbf{A}_s^* is the Hermitian of \mathbf{A}_s

III. RECONSTRUCTION ALGORITHM

The performance of CTF block depends on the reconstruction algorithm used. A non-linear algorithm is required to get the support. There are many algorithms like convex iterative algorithms, greedy algorithms and Bayesian algorithms. Basis Pursuit is one of the standard algorithm which employs l_1 optimization. And some algorithms namely Subspace pursuit, Compressive sensing matching pursuit utilizes backtracking method. Compared to all, greedy algorithms are faster iterative algorithms which include Matching pursuit, Orthogonal matching pursuit. This uses the idea of finding the location of maximum energy atom. Moreover there are many derivative algorithms of OMP has emerged namely StagewiseOMP [8], RegularizedOMP [9], Stagewise weakOMP. As these are iterative algorithms, the number of measurements required is increased for perfect recovery. However complexity for reconstructing the signal is very less compared with the standard l_1 norm optimization.



A sparse signal can be recovered using greedy algorithms. It identifies the support set iteratively. The standard greedy algorithm is Orthogonal Matching Pursuit (OMP) and of course many modifications were developed [5] in view of increasing the performance. Orthogonal Matching Pursuit recovers k unknown values of k -sparse signal. OMP initializes the residual with the measurement matrix and selects the best (maximum) value from the inner product of A and residual. Then updating the residual and repeating to find the best values continues until k iterations or the residual norm is less than threshold. Finally, the location columns act as the support set and the corresponding values as reconstructed signal. In CTF block, the support set is considered and recovered signal is obtained using Eq.(3).

In this paper different greedy algorithms are used. The Simultaneous Orthogonal Matching Pursuit is same as OMP but the inner product is the correlation vector. Instead of running k iterations, since the residual may not be zero, SOMP is repeated to few more iterations [6]. This lead to a derived algorithm-EOMP. Assuming an extra run factor α which varies from 0 to 1, the iterations are modified as $t = K + \lfloor \alpha K \rfloor$. The pseudo-code of Simultaneous orthogonal matching pursuit algorithm is presented as below:

Algorithm1 : Simultaneous Orthogonal Matching Pursuit

Input: A $m \times L$ pseudo-random matrix A , $m \times 2K$ frame vector V and the number of sub bands K and iteration counter $t=1$.

Output: Support set Λ which is an $1 \times 2K$ vector.

1) Initialize: a null vector $\Lambda = \emptyset$ and the residual $res = V$.

2) Get the projection of residual over the measurement matrix and find the maximum location of this projection as

$$\lambda_t = \arg \max_{j=1,2,\dots,L} \left| \left\langle res_{t-1}, A_j \right\rangle \right| / d_k$$

where d_k is the norm of diagonal elements of A .

3) obtain the symmetric location and merge

$$\Lambda_t = \Lambda_{t-1} \cup \lambda_t \cup \lambda_{tsymm}$$

4) Least squares problem can be solved to estimate the

$$\text{signal as } \hat{x} = A_{\Lambda_t}^\dagger V, \text{ where } A^\dagger = (A^T A)^T A$$

5) update the residual as $res_t = V - A_{\Lambda_t} \hat{x}$ and

$$resnorm = \|res\|_2$$

6) If $t < K$ and $resnorm < threshold$, go to step 2; else terminate.

7) Estimated signal with support set Λ_t is calculated.

SOMP is modified form of OMP and achieves faster recovery. SOMP may fail in selecting the exact support set in K iterations as the residual may not be zero. Hence extending the iterations beyond K to get correct support lead to enhanced SOMP. Repeat from step-2 until the iterations reach the criterion $t = k + \lfloor \alpha k \rfloor$, where k is the sparsity of the signal and $\alpha \in [0,1]$ is the extended run factor.

Algorithm 2: Enhanced Simultaneous Orthogonal Matching Pursuit

Input: A $m \times L$ pseudo-random matrix A , $m \times 2K$ frame vector V and the number of sub bands K and iteration counter $t=1$, enhanced parameter $\alpha \in [0,1]$.

Output: Support set Λ which is an $1 \times 2K$ vector.

1) Initialize: a null vector $\Lambda = \emptyset$ and the residual $res = V$.

2) Get the projection of residual over the measurement matrix and find the maximum location of this projection as

$$\lambda_t = \arg \max_{j=1,2,\dots,L} \left| \left\langle res_{t-1}, A_j \right\rangle \right| / d_k$$

where d_k is the norm of diagonal elements of A .

3) obtain the symmetric location and merge

$$\Lambda_t = \Lambda_{t-1} \cup \lambda_t \cup \lambda_{tsymm}$$

4) Least squares problem can be solved to estimate the

$$\text{signal as } \hat{x} = A_{\Lambda_t}^\dagger V, \text{ where } A^\dagger = (A^T A)^T A$$

5) update the residual as $res_t = V - A_{\Lambda_t} \hat{x}$ and

$$resnorm = \|res\|_2$$

6) If $t < k + \lfloor \alpha k \rfloor$ and $resnorm < threshold$, go to step 2; else terminate.

7) Estimated signal with support set Λ_t is calculated.

Thus by extending the iterations, enhanced SOMP improved the chance of finding the correct atoms. When $\alpha = 0$ leads to the standard OMP performance. Prior knowledge of sparsity is the only drawback of OMP even though its performance is superior to all other greedy algorithms. When the restriction on iterations is removed, then knowledge of sparsity is not necessary. Another modified OMP was proposed by continuing the iterations until residue becomes zero.

IV. SIMULATION RESULTS

To compare the performances of EOMP and OMP algorithms, 500 monte-carlo simulations were done for each algorithm. A multiband signal $x(t)$ is considered which has 3 frequency bands i.e., $N = 2N_0 = 6$, each has a bandwidth of 50 MHz. Since it is a wideband signal, its sampling frequency is around 10 GHz. This signal is applied to MWC with 100 parallel channels and modulated with a carrier frequency of $[0 - 5GHz]$ and the spectrum occupancy $NB = 300MHz$, thus $NB \ll f_{NYQ}$. Aliasing rate taken is 195 whereby the sampling frequency f_s which is equal to f_p , the rate of pseudo-random generator is given by



$f_{NYQ} / 195 = 51.3\text{MHz}$. With this parameter setting; the spectral support can be recovered by using both SOMP and enhanced OMP reconstruction algorithms as shown in fig. 4 and fig.5. The multiband spectrum with carrier frequency $f_i = [3\text{GHz}, 3.1\text{GHz}, 1.4\text{GHz}]$ was recovered which indicates the occupied spectrum and remaining spectrum is vacant.

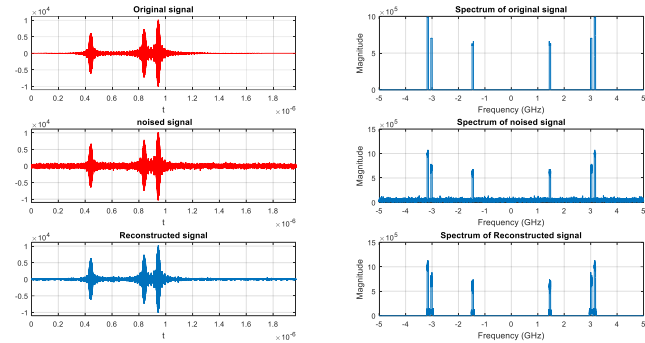


Fig 4: Multiband Signal and spectrum of Original, noised and reconstructed signals using SOMP algorithm

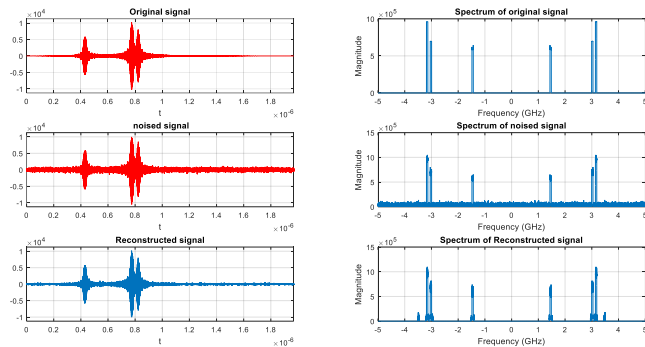


Fig 5: Multiband Signal and spectrum of Original, noised and reconstructed signals using Enhanced SOMP algorithm

As depicted if Fig.6, there is an improvement in the support recovery as the number of bands are increasing with enhanced OMP compared to SOMP. This makes CR to identify the active bands and allot to the secondary users.

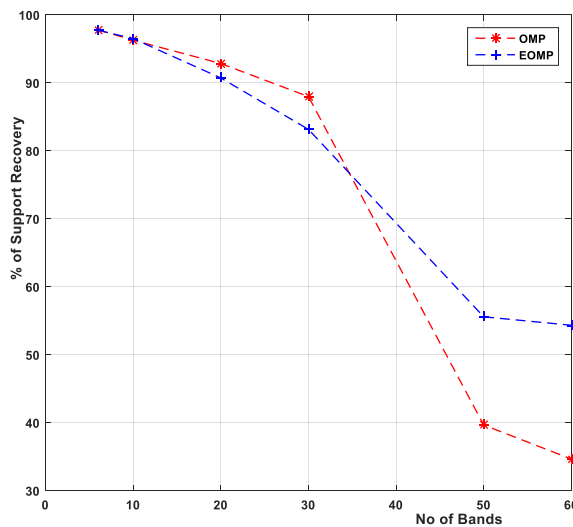


Fig 6: Percentage of Support recovery vs SNR

The Normalized Residual of EOMP is lower than OMP as SNR is increasing. In the OMP and SOMP algorithm, the number of iterations depends on the sparsity. Some of the

atoms may not be converged. EOMP runs the iterations still further to reduce the residue which is as depicted in Fig 7.

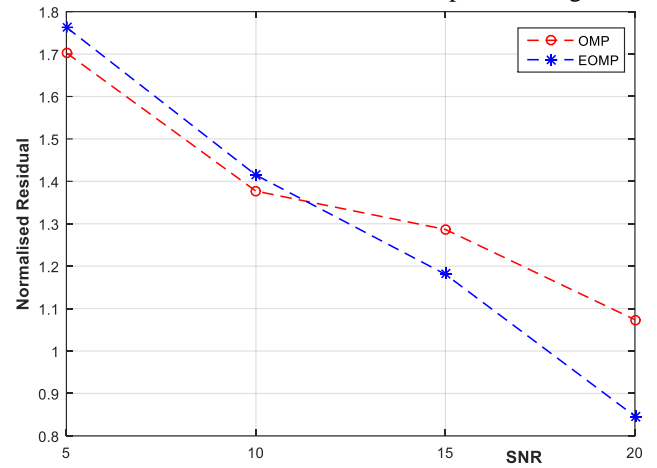


Fig 7: Normalised Residual vs SNR

Performance analysis of these algorithms was done by another metric named mean square error which is given by Eq.4. Also the benefits of MSE are maintained in various SNRs shown in Fig 8. Better MSE performance is achieved for all algorithms by increasing SNR

$$\text{Normalized } MSE = \frac{\|\hat{x} - x\|_2}{\|x\|_2} \quad (4)$$

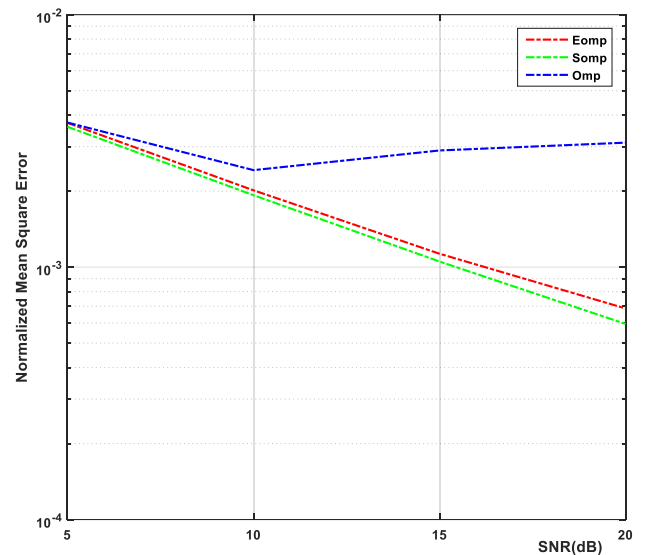


Fig 8: MSE performance for the algorithms

Monte Carlo simulation for 100 trials was done to compare the performance of these algorithms. Better MSE performance is achieved for all algorithms at increasing SNR. We use $M=195$, $m=50$ channels and SNR is varied from 5 dB to 20 dB. The Normalised MSE at different SNR is depicted in Fig.8. The performance for SOMP with

$MSE \approx 5.95 \cdot 10^{-4}$ is almost equal for Enhanced OMP with $MSE \approx 6.85 \cdot 10^{-4}$ and compared with standard OMP with $MSE \approx 0.31 \cdot 10^{-3}$ at SNR of 20 dB.

The results shown in Fig.9. depicts the probability of successfully recovering the support at various number of channels. At each channel periodic mixing of random sequence with input signal $x(t)$ was applied. The recovery support reaches at least 90% at SNR above 10dB and minimum 30 channels of usage. Also still the improvement is high with enhanced SOMP algorithm.

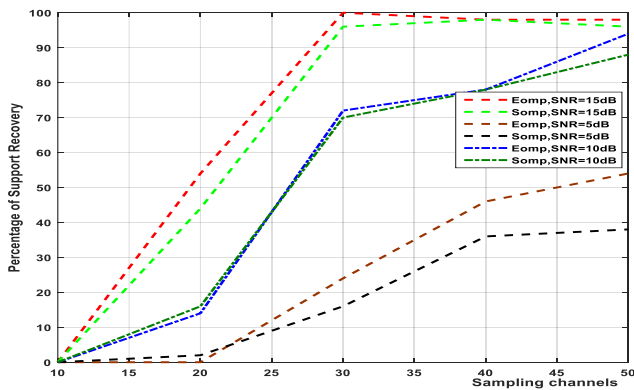


Fig.9: Reconstruction Probability at varying channels for OMP and its derived algorithms when SNR =5 dB, 10 dB, 15 dB

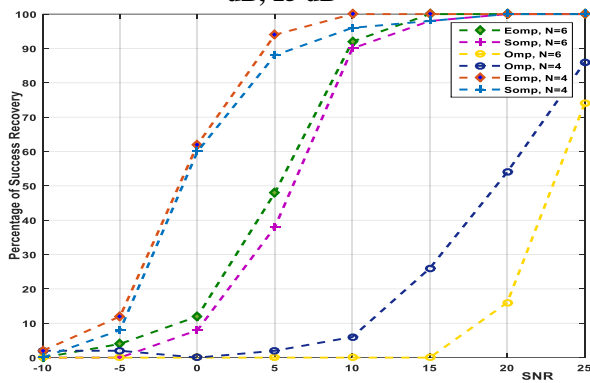


Fig.10: Reconstruction Probability at different SNR for OMP and its derived algorithms when N=4 and N=6.

As from the simulations, Fig.10 shows the improvement of enhanced OMP since the iterations are increased. Success recovery is improved even at low SNR. Thus MWC improved the sensing capabilities of cognitive radio by sensing the spectrum at faster rates. Fig.11 depicts the enhanced SOMP for different values of $\alpha = [0, 1]$. Fig.12 shows the runtime of enhanced SOMP is vastly improved with that of standard OMP.

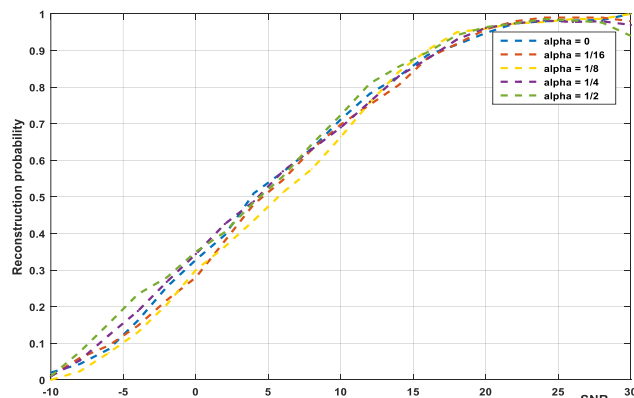


Fig.11. Reconstruction Probability vs SNR for Enhanced SOMP with different values of run factor α

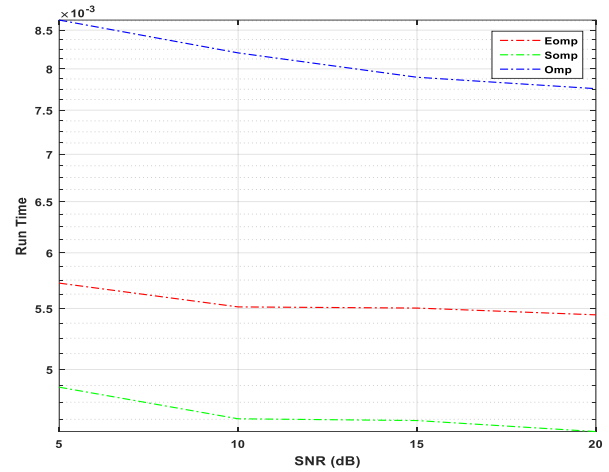


Fig.12. Recovery Run time vs SNR for m=50 channels and N=6

V. CONCLUSION

In this work, sub-Nyquist sampling of wide band spectrum using MWC architecture was considered. Greedy algorithms are advantageous in terms of computational costs. To increase the performance, instead of OMP, its derivative algorithms were used. The simulation results proved the improvement of reconstruction probability using enhanced OMP at an increase of the bands. In addition, the improved algorithm also suitable at low SNRs. Further, the proposed algorithms identify the correct support at increase in number of iterations and also no knowledge of sparsity is required. Thus reconstruction with enhanced SOMP outperforms the conventional MWCs. Also the benefits of MSE are maintained at various SNRs.

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